Math 121 Chapter 1: Functions

1.1 Real Numbers, Inequalities, and Lines.

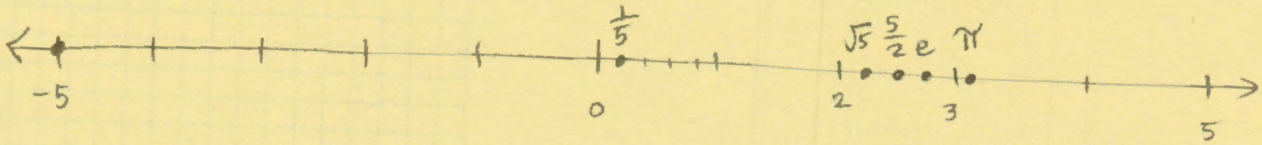
Objectives

1) Use inequalities to describe the relative size of real numbers
2) Use set notation and interval notation to describe finite and infinite intervals.
3) Find and interpret the slope of a line
4) Write equations of lines using
   - slope-intercept form $y=mx+b$
   - point-slope formula $y-y_1=m(x-x_1)$
5) Write equations and find slopes of
   - vertical lines
   - horizontal lines
6) Use the general form of the equation of a line
   $ax+by=c$ where $a>0$, $a,b\&c$ integers
7) Find and interpret the y-intercept of a line.
8) Find equations of lines
   - parallel to a given line
   - perpendicular to a given line
9) Find a linear regression model given a set of data and use it to predict trends

① Plot the following numbers on a number line

$\{-5, \frac{1}{5}, \frac{5}{2}, \sqrt{5}, \pi, e\}$

$\approx \{-5, .2, 2.5, 2.24, 3.14, 2.72\}$



② Write the appropriate symbol < or > between:

$\frac{1}{5} > -5$

$\sqrt{5} > \frac{1}{5}$

$\sqrt{5} < e$

③ Complete the missing notation.

Interval Notation	Set Notation	Graph	Type
$[-2,5]$	$\{x \mid -2 \leq x \leq 5\}$	-2 5	finite closed
$(-2,5)$	$\{x \mid -2 < x < 5\}$	-2 5	finite open
$[-2,5)$	$\{x \mid -2 \leq x < 5\}$	-2 5	finite half-open or half-closed
$(-2,5]$	$\{x \mid -2 < x \leq 5\}$	-2 5	finite half-open or half-closed
$[-2,\infty)$	$\{x \mid x \geq -2\}$	-2	infinite closed
$(-2,\infty)$	$\{x \mid x > -2\}$	-2	infinite open
$(-\infty,-2]$	$\{x \mid x \leq -2\}$	-2	infinite closed
$(-\infty,-2)$	$\{x \mid x < -2\}$	-2	infinite open

(4) Write equation of each line.
Write answer in general form.

a) slope 5, y intercept $\frac{1}{5}$.

b) slope 5, passing through $(1,\frac{1}{5})$

c) vertical, passing through $(1,\frac{1}{5})$

d) horizontal, passing through $(1,\frac{1}{5})$

e) passing through $(2,5)$ and $(-\frac{1}{5},\frac{1}{2})$

f) parallel to $y=5x-3$, passing through $(1,\frac{1}{5})$

g) perpendicular to $y=5x-3$, passing through $(1,\frac{1}{5})$.

a) $m=5, b=\frac{1}{5}$ $\quad y=5x+\frac{1}{5}$

$5y=25x+1$ clear fractions

$-1=25x-5y$

$\boxed{25x-5y=-1}$ general form.

b) $m=5, \ (1,\frac{1}{5})$

$y-y_1=m(x-x_1)$

$y-\frac{1}{5}=5(x-1)$

$y-\frac{1}{5}=5x-5$

$5y-1=25x-25$

$24=25x-5y$

$\boxed{25x-5y=24}$

c) $\boxed{x=1}$ x coordinate

d) $y=\frac{1}{5}$

$\boxed{5y=1}$ y coordinate

e) $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{5 - \frac{1}{2}}{2 - \frac{-1}{5}} = \frac{\frac{9}{2}}{\frac{11}{5}} = \frac{9}{2} \cdot \frac{5}{11} = \frac{45}{22}$$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{45}{22}(x - 2)$$

$$22y - 110 = 45x - 90$$

$$-20 = 45x - 22y$$

$$\boxed{45x - 22y = -20}$$

f) parallel means <u>same</u> slope

slope of $y = 5x - 3$ is $m = 5$

through $(1, \frac{1}{5})$

$$y - \frac{1}{5} = 5(x - 1)$$

$$5y - 1 = 25(x - 1)$$

$$5y - 1 = 25x - 25$$

$$24 = 25x - 5y$$

$$\boxed{25x - 5y = 24}$$

g) perpendicular means slope is <u>opposite</u> and <u>reciprocal</u>

new slope $m = -\frac{1}{5}$

$$y - \frac{1}{5} = -\frac{1}{5}(x - 1)$$

$$5y - 1 = -x + 1$$

$$\boxed{x + 5y = 2}$$

(5) Given the following data:

Birth year	1970	1980	1990	2000	2010
Life expectancy (♀)	74.7	77.4	78.8	79.3	80.8

a) Find the line of best fit by using linear regression on your calculator. Use x=1 for 1970, x=2 for 1980, etc.

On your GC:

[2nd] [(] means {	use { to start a list
[,] is comma	use , to separate numbers in the list
[2nd] [)] means }	use } to end a list
[STO>] means "Store"	use store to place a list in memory
[2nd] [1] means L1	use L1 as the list of values of x.
[2nd] [2] means L2	use L2 as the list of values of y.

Your calculator automatically uses L1 and L2 for regression.

<u>step1</u>: Store the data in L1 and L2

[2nd] [(] 1 [,] 2 [,] 3 [,] 4 [,] 5 [2nd] [)] [STO>] [2nd] 1 [ENTER]

$\{1, 2, 3, 4, 5\} \rightarrow L1$

[2nd] [(] 74.7 [,] 77.4 [,] 78.8 [,] 79.3 [,] 80.8 [2nd] [)] [STO>] [2nd] 2 [ENTER]

$\{74.7, 77.4, 78.8, 79.3, 80.8\} \rightarrow L_2$

<u>step2</u>: Calculate the linear regression model.

[STAT] ▷ 4

EDIT [CALC] TESTS
4. LinReg (ax+b)

[ENTER]

LinReg(ax+b)

LinReg (ax+b)
y=ax+b
a= 1.31
b= 74.17

Answer:
The line of best fit is $y = 1.31x + 74.17$

b) Graph the data and the regression line on your GC.

Step 1: Graph the data using STATPLOT

[2nd] [Y=] means STATPLOT

1. select plot1.

[ENTER]

[Plot1]

[On] Off

Turn on data plotting.

[WINDOW]

XMIN 0
XMAX 6
XSCL 1

YMIN 0
YMAX 90
YSCL 10

[2nd] [MODE] means QUIT.

Step 2: Graph the regression line.

[Y=]

Y_1 [CLEAR]

[VARS]

5

[VARS] Y-VARS

5. Statistics

▷ ▷ to EQ

1 or ENTER

XY Σ [EQ] TEST PTS

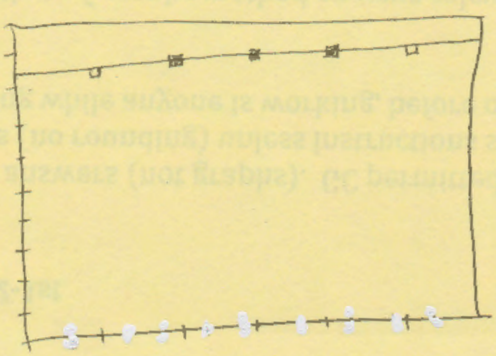
[1.] RegEQ

RegEQ is a memory location which contains the most recent regression.

[Plot1]

$Y_1 = 1.31x + 74.17$

[GRAPH]



c) Use the $\boxed{\text{TABLE}}$ to compare the data to the regression line.

$\boxed{\text{2nd}}$ $\boxed{\text{WINDOW}}$ is $\boxed{\text{TBLSET}}$

0 $\boxed{\text{ENTER}}$
1 $\boxed{\text{ENTER}}$
$\boxed{\text{ENTER}}$
▽
$\boxed{\text{ENTER}}$

TABLE SETUP
TblStart = 0
ΔTbl = 1
Indpnt: $\boxed{\text{AUTO}}$ Ask
Depend: $\boxed{\text{AUTO}}$ Ask

$\boxed{\text{2nd}}$ $\boxed{\text{GRAPH}}$ is $\boxed{\text{TABLE}}$

X	Y_1	vs.	Data	Regression is...
0	74.17			
1	75.48		74.7	a bit high
2	76.79		77.4	a bit low
3	78.1		78.8	a tiny bit low
4	79.41		79.3	a tiny bit high
5	80.72		80.8	a tiny bit low

d) Interpret the slope and y-intercept of the regression line.

$y = 1.31x + 74.17$

$m = \frac{\Delta y}{\Delta x} = \frac{1.31 \text{ years exp.}}{1 \text{ decade}}$ y is life expectancy in years
x is... decades... where x=0 is 1960.

Interpretation: of slope — Life expectancy increases by 1.31 each decade (or 10 years) that passes.

OR 1.31 years per decade.

Better interpretation:

$m = \frac{1.31 \text{ years expected}}{10 \text{ years}} = \frac{.131 \text{ years expected}}{1 \text{ year elapsed}}$

Interpretation of slope: Life expectancy increases by .131 year per year.

y-intercept: 74.17

when $x=0$, $y=74.17$

$x=0$ means 1960.

Interpretation: of y-int	In 1960, the regression line predicts that life expectancy was 74.17 years.

e) Find and interpret the x-intercept of the regression line.

set $y=0$, solve for x

$y = 1.31x + 74.17$

$0 = 1.31x + 74.17$

$-74.17 = 1.31x$

$-\frac{74.17}{1.31} = x$

$-\frac{7417}{131} = x \approx -56.6$ or -57

Slightly less that 57 decades before 1960

means $\approx$ 566 years before 1960.

means 1394.

In the year 1394, the life expectancy was zero.

Does this make sense? <u>No!</u> In 1394, ~~people~~ women still lived to reproductive age (on average), or people would have become extinct!

<u>The Moral of this story</u>: Linear trends, as captured by a linear regression model, may <u>not</u> be valid predictors a long way before or a long way after the data used to find the model.

f) Use the regression line to predict life expectancy in 2025.

$2025 - 1960 = 65$ years $= 6.5$ decades after 1960.

Substitute $x=6.5$

$y = 1.31x + 74.17$

$y = 1.31(6.5) + 74.17$

$\boxed{y = 82.685 \text{ years expected in 2025}}$